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Conjugate heat transfer from a translating drop in an electric field at low Peclet number

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Abstract—The conjugate heat exchange with transient interfacial temperature between a translating liquid drop and its host fluid in a uniform electric field is considered. Singular perturbation is developed to obtain the temperature within the domain of the continuous phase whereas regular perturbation is used to obtain the solution inside the drop with the help of the method of weighted residuals. This method proves to be powerful for the solution of problems with time-dependent non-homogeneities arising within the governing equation and/or the boundary conditions. The temperature is computed up to and including the first order in the Peclet number; however, higher order is also performed for the host phase in order to examine the influence of an external field upon the total transport rates. In the first order solution, the effects of an electric field were to alter the temperature inside and outside the droplet as well as the heat flux, but the net heat transfer rate, which is totally controlled by conduction and convection, remains unchanged. Beyond the first order approximation, the contribution to the net heat transfer due to the electric field becomes assessable.

1. INTRODUCTION

THE APPLICATION of an electric field to transport processes offers several advantages. Among them are the surface area generation by means of droplet rupture [1], and the increase of the transport rates through the electric field-induced motion both inside and outside the drop or the electrically forced oscillation of the drop itself [2, 3]. Although the augmentation of heat transfer in liquids has been observed in the experiment of Kronig and Ahsmann [4] nearly 40 years ago, very little is understood about its enhancement mechanisms.

Not until recently has the problem received a considerable renewed interest motivated by a desire to develop compact direct-contact liquid heat exchangers for space applications, and to resolve some technical difficulties related to traditional mechanical agitations as commonly found in numerous chemical engineering processes. As a result, a number of studies have been conducted in the past few years in an effort to understand the phenomena. For a complete update of the activities in this area, the readers are referred to an authorative review by Jones [5]. Due to the diverse nature of the problem, we will restrict ourselves to the type of motion known as the Taylor flow in that the electrical stress is the sole electrohydrodynamic coupling. To date none of the theoretical investigations has produced a satisfactory picture of the transfer behavior in the presence of an external electric field. Within the scope of this context, Morrison [6], Griffiths and Morrison [7, 8] and Sharpe and Morrison [9] treated the situation as an external problem with controlling resistance being in the continuous phase. This is equivalent to the case where there exists a fictitious mixing mechanism to destroy the temperature gradient within the drop interior. On the opposite extreme, Chung and Oliver [10] studied the transport aspects of the internal problem in which the host phase is completely isolated from the dispersed phase. According to Abramzon and Borde [11], the criteria for those limitations may be determined from the contribution of each phase to the total heat transfer resistance which may roughly be calculated from the characteristic cooling/heating time for each phase, the ratio of which gives an estimate of the relative magnitude of phase resistances:

$$\frac{\hat{t}_c}{t_c} \cong 0.033\Phi_\kappa Nu_{ss} \qquad (1)$$

where Φ_κ is the thermal conductivity ratio of the continuous to dispersed phase, and Nu_{ss} is the steady state Nusselt number. For liquids of similar properties, this ratio is of the order of unity; therefore neither one of the two approaches discussed above is adequate. Under this situation we encounter the so-called 'conjugated' problem in that the governing equations of heat transfer for both the continuous and dispersed phases have to be solved simultaneously. Chang *et al.* [12] and Chang and Berg [13] attempted to solve this problem in an approximate manner by using the thin boundary layer approximations to neglect the molecular diffusion in the θ direction. Such simplification is justified only during a short period of exposure, and is invalid as time increases due to the growth of the boundary layer with time.

In this paper we concentrate our efforts to pose the problem of heat transfer associated with a drop translating in an electric field in a more realistic term,

NOMENCLATURE

A	constant coefficients in the velocity equation (4)
$\hat{A}$	see equation (41)
B_n	constant in equation (13)
C_n	constants, see equations (29)
E	electric field strength
f_{ij}	unknown function defined by equation (19)
g_k	see equation (8a)
G_k	see equation (8b)
$K_{n+1/2}$	spherical Bessel function
Nu	steady state Nusselt number
P	Legendre polynomial
Pe	Peclet number
Q	heat transfer rate
Q''	heat flux
R	drop radius
R_j	see equation (27)
r	dimensionless radial coordinate
$\widehat{RCC}$	see definition (44)
RHS	right-hand side, equation (20)
$\hat{\mathcal{R}}$	residual
T	dimensional temperature
t_c	characteristic time
$\mathbf{U}$	velocity vector
W	dimensionless field strength
Z	dimensionless temperature.

Greek symbols

α	thermal diffusivity
β_i^j	see Appendix B
γ	Euler–Mascheroni constant
ε	perturbation parameter
θ	angular coordinate
κ	thermal conductivity
$\bar{\mu}$	$\cos\theta$
ρ	density
τ	dimensionless time
Φ_x	ratio of property x of the continuous to dispersed phase
ω_i^j	see Appendix A.

Subscripts

s	surface condition
ss	steady state condition.

Superscripts

in	inner solution
out	outer solution.

Overheads

$\hat{}$	dispersed phase.

then develop a singular perturbation procedure for the conjugate problem. In addition to the conjugative aspects of the problem, it differs from the work of Griffiths and Morrison [7] in that the present formulation also includes the translational velocity of the drop for which the regular perturbation expansion fails to yield a uniform solution. Furthermore it also takes into account the peripheral variations of the interfacial temperature.

2. FORMULATION

Consider a pure liquid drop at temperature T_0, translating with a terminal velocity U_∞ in another immiscible liquid of infinite extent, held at temperature T_∞, under the influence of a uniform electric field whose strength E is assumed not very strong in order to prevent a charge leakage from the two-phase system. Due to the temperature difference, heat will flow from/to the drop depending on the direction of temperature gradient as dictated by the second law of thermodynamics. This heat transfer process can be described by the energy equation written in dimensionless form as

$$Pe\,\mathbf{U}\cdot\nabla Z = \nabla^2 Z \tag{2}$$

$$\frac{\partial \hat{Z}}{\partial \tau} + \hat{P}e\,\hat{\mathbf{U}}\cdot\nabla\hat{Z} = \nabla^2\hat{Z} \tag{3}$$

where $Pe\,(=U_\infty R/\alpha)$ is the Peclet number for the continuous phase with α being the thermal diffusivity, $\hat{P}e\,(=U_\infty R/\hat{\alpha})$ the Peclet number for the drop phase, $\tau(=t\alpha/R^2)$ the dimensionless time, $\mathbf{U}=(U_r, U_\theta)$ is the velocity vector, and Z is the local dimensionless temperature defined by $(T-T_\infty)/(T_0-T_\infty)$. In the above equations, the transport within the dispersed phase, distinguished from the continuous phase by a 'hat', is treated as transient because the steady state cannot be attained unless the drop is in thermal equilibrium with its surroundings. The electrohydrodynamic problem in the creeping regime has been solved by superposing Taylor and Hadamard-Rybczynski flows [12], and the results are quoted for completeness:

$$U_r(r,\theta) = \left(\frac{2A_1}{r^3}+\frac{2A_2}{r}+1\right)\cos\theta + A_3\left(\frac{1}{r^2}-\frac{1}{r^4}\right)(3\cos^2\theta-1) \tag{4a}$$

$$U_\theta(r,\theta) = \left(\frac{A_1}{r^3}-\frac{A_2}{r}-1\right)\sin\theta - \frac{2A_3}{r^4}\sin\theta\cos\theta \tag{4b}$$

$$\hat{U}_r(r,\theta) = A_4(2-2r^2)\cos\theta - A_3(r-r^3)(3\cos^2\theta-1) \tag{4c}$$

$$\hat{U}_\theta(r,\theta) = A_4(4r^2-2)\sin\theta - A_3(5r^3-3r)\sin\theta\cos\theta \tag{4d}$$

where the constant coefficient A values are defined by the following expressions: $A_1 \equiv 1/4(1+\Phi_\mu)$, $A_2 \equiv -(3+2\Phi_\mu)/4(1+\Phi_\mu)$, $A_3 \equiv W/4(1+\Phi_\mu)$, and $A_4 \equiv -\Phi_\mu/4(1+\Phi_\mu)$. Also, r is the dimensionless radial coordinate, θ is the angular coordinate, $W = 4V_{cr}(1+\Phi_\mu)/U_\infty$ is the measure of the relative importance of the electric field to drop translation and may be interpreted as the dimensionless field strength. V_{cr} introduced in the definition of W is the maximum velocity generated by the electric field in the absence of the translational motion, and has a complicated functional dependency upon the electrical properties [12].

In order to make the problem well-posed, we specify the following boundary conditions that include the appropriate limit far away from the drop surface, the continuity of temperature and heat flux at the interface, i.e.

$$\Phi_\kappa \frac{\partial Z}{\partial r}(1,\bar{\mu}) = \frac{\partial \hat{Z}}{\partial r}(1,\bar{\mu}) \tag{5a}$$

$$Z(1,\bar{\mu}) = \hat{Z}(1,\bar{\mu}) = Z_s(\tau,\bar{\mu}) \tag{5b}$$

where Z_s is the interfacial temperature with angular dependence. Here it is modeled by two terms: the first term is for spherically symmetric conduction whereas the second is added to account for the effects of convective motions due to both the translation and the electric field

$$Z_s(\bar{\mu}) = Z_{00}+\varepsilon[Z_{01}P_0(\bar{\mu})+Z_{11}P_1(\bar{\mu})+Z_{21}P_2(\bar{\mu})] \tag{6}$$

in which $\bar{\mu}$ represents $\cos\theta$, ε is the Peclet number for the continuous phase and is used as a perturbed parameter, and P_i is the Legendre polynomial of order i. Other constraints that assure the symmetry of the problem about the x-axis will be imposed in the next section.

3. METHOD OF SOLUTION

Due to the presence of the translational motion of the drop, regular perturbation [7] is of limited use because the solution does not approach the common limit far away from the surface. Physically this means that there exists a region where the conduction is no longer dominant as illustrated from the ratio of the convection to conduction flux

$$Pe\frac{\mathbf{U}\cdot\nabla Z}{\nabla^2 Z} = O(r\,Pe) \tag{7}$$

which clearly shows that conduction is the major mode of heat transfer near the drop surface whereas convection dominates far from the drop regardless of how small Pe may be.

To overcome the deficiency of the traditional perturbation series, it is necessary to employ a singular perturbation method in that the inner and outer solutions are expanded in the following forms:

$$Z^{\text{in}}(r,\bar{\mu}) = \sum_{k=0}^{\infty} g_k(\varepsilon)Z_k^{\text{in}}(r,\bar{\mu}) \tag{8a}$$

$$Z^{\text{out}}(\rho,\bar{\mu}) = \sum_{k=0}^{\infty} G_k(\varepsilon)Z_k^{\text{out}}(\rho,\bar{\mu}) \tag{8b}$$

where ρ is regarded as the rescaling coordinate ($\rho = \varepsilon r$, $\varepsilon = Pe$), and the functions $g_k(\varepsilon)$ and $G_k(\varepsilon)$ are subjected to the restrictions

$$\lim_{\varepsilon\to 0}\frac{g_{k+1}}{g_k} = 0 \quad \text{and} \quad \lim_{\varepsilon\to 0}\frac{G_{k+1}}{G_k} = 0. \tag{9}$$

To complete the solution we must match the inner solution at its 'farthest extremity' with the outer solution at its 'nearest extremity' asymptotically.

3.1. *Leading order solution*

We now proceed to construct the solutions by substituting equation (7) into equation (2) with the assumption that $g_0(\varepsilon) = 1$. The leading order equations are obtained, by equating the coefficients of ε^0, as

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}Z_0^{\text{in}}}{\mathrm{d}r}\right) = 0 \tag{10}$$

and the integration of equation (10) leads to a solution that satisfies the appropriate boundary condition at the interface as

$$Z_0^{\text{in}}(r) = Z_{00}+C_1\left(1-\frac{1}{r}\right) \tag{11}$$

where C_1 is the constant of integration to be determined shortly.

For the outer region, the repetition of the above steps gives the leading order governing equation in terms of the restrained coordinate by

$$\frac{1}{\rho}\left\{\frac{\partial}{\partial\rho}\left(\rho^2\frac{\partial Z_0^{\text{out}}}{\partial\rho}\right)+\frac{\partial}{\partial\bar{\mu}}\left[(1-\bar{\mu}^2)\frac{\partial Z_0^{\text{out}}}{\partial\bar{\mu}}\right]\right\} = \bar{\mu}\frac{\partial Z_0^{\text{out}}}{\partial\rho}+\frac{(1-\bar{\mu}^2)}{\rho}\frac{Z_0^{\text{out}}}{\partial\bar{\mu}}. \tag{12}$$

In a different context, Acrivos and Taylor [14] obtained an analytical solution to equations (12) in the form of a series of product of spherical Bessel and Legendre functions

$$Z_0^{\text{out}}(\rho,\bar{\mu}) = \mathrm{e}^{\rho/2\bar{\mu}}\sqrt{\left(\frac{\pi}{\rho}\right)}\sum_{n=0}^{\infty} B_n K_{n+1/2}(\rho/2)P_n(\bar{\mu}) \tag{13}$$

where B_n are constants, and the spherical Bessel function is defined as

$$K_{n+1/2}(\rho/2) = \mathrm{e}^{-\rho/2}\sqrt{\left(\frac{\pi}{\rho}\right)}\sum_{i=0}^{n}\frac{(i+n)!}{i!(n-i)!\rho^i}. \tag{14}$$

The constants in equations (11) and (13) are now calculated from the matching principle that demands the two solutions to approach the common limit in the overlapping region. Mathematically, this requirement may be stated as

$$\lim_{r\to\infty} Z_0^{\text{in}}(r,\bar{\mu}) = \lim_{\rho\to 0} G_0(\varepsilon) Z_0^{\text{out}}(\rho,\bar{\mu}). \quad (15)$$

For expression (15) to be met, the complete leading order solutions with $G_0(\varepsilon) = \varepsilon$ reduce to

$$Z_0^{\text{in}}(r) = \frac{Z_{00}}{r} \quad (16)$$

$$Z_0^{\text{out}}(\rho,\bar{\mu}) = \frac{Z_{00}}{\rho} e^{-\rho/2(1-\bar{\mu})}. \quad (17)$$

3.2. *Higher order approximations*

For higher order solutions, the algebra in the matching process becomes tedious, without even mentioning the complexities introduced by the governing equations. However, the next higher order solutions may be obtained with a reasonable amount of effort. Due to the lengthy algebraic involvement and the straightforwardness of the procedure, we only briefly outline the steps.

The nth order equation can be written in the following form:

$$\frac{1}{r^2}\left\{\frac{\partial}{\partial r}\left(r^2\frac{\partial Z_n^{\text{in}}}{\partial r}\right)+\frac{\partial}{\partial \bar{\mu}}\left[(1-\bar{\mu}^2)\frac{\partial Z_n^{\text{in}}}{\partial \bar{\mu}}\right]\right\}$$
$$= U_r\frac{\partial Z_{n-1}^{\text{in}}}{\partial r} - U_\theta\frac{(1-\bar{\mu}^2)^{1/2}}{r}\frac{\partial Z_{n-1}^{\text{in}}}{\partial \bar{\mu}}. \quad (18)$$

Upon inspection of equations (4a, b), (6), and (18), it would appear that a portion of equation (18) may be simplified if the solutions are sought in the form of a linear combination of the products of unknown functions of r and the first $2n$ Legendre polynomials, i.e.

$$Z_n^{\text{in}}(r,\bar{\mu}) = \sum_{j=0}^{2n} f_{jn}(r)P_j(\bar{\mu}) \quad (19)$$

where the subscripts j and n indicate the series index and order of approximation respectively.

The advantage of expressing the solutions as equation (19) comes to light from the fact that it allows one to solve equation (18) termwise. More importantly, it transforms the partial differential equation into ordinary differential equations which are certainly much easier to deal with since their solution techniques are well developed. For $n = 1$, the right-hand side of equation (18) can be evaluated from the leading order solution (16) and the velocity of the continuous phase (4a, b), then using the fact that a polynomial of power i can be fitted exactly by the first $i+1$ Legendre functions, one can equate terms containing like order Legendre polynomials so that RHS_1^{in} may be written as

$$\text{RHS}_1^{\text{in}} = -Z_{00}\left(2\frac{A_1}{r^5}+2\frac{A_2}{r^3}+\frac{1}{r^2}\right)P_1(\bar{\mu})$$
$$+2A_3Z_{00}\left(\frac{1}{r^6}-\frac{1}{r^4}\right)P_2(\bar{\mu}). \quad (20)$$

Combining equations (18)–(20) yields the Euler equations whose solutions are readily solved using the method of undetermined coefficients

$$f_{01}(r) = \frac{Z_{01}}{r} - \frac{Z_{00}}{2}\left(1-\frac{1}{r}\right) \quad (21a)$$

$$f_{11}(r) = \frac{Z_{11}}{r^2} - Z_{00}\left[\frac{A_1}{2}\left(\frac{1}{r^3}-\frac{1}{r^2}\right)+A_2\left(\frac{1}{r^2}-\frac{1}{r}\right)+\frac{1}{2}\left(\frac{1}{r^2}-1\right)\right] \quad (21b)$$

$$f_{21}(r) = \frac{Z_{21}}{r^3}+\frac{1}{6}A_3Z_{00}\left(\frac{2}{r^4}-\frac{5}{r^3}+\frac{3}{r^2}\right) \quad (21c)$$

with the help of the matching condition

$$\lim_{r\to\infty}[Z_0^{\text{in}}(r,\bar{\mu})+\varepsilon Z_1^{\text{in}}(r,\bar{\mu})] = \lim_{\rho\to 0}[G_0(\varepsilon)Z_0^{\text{out}}(\rho,\bar{\mu})]. \quad (22)$$

We now turn our attention to the next term of the outer solution described by

$$\frac{1}{\rho}\left\{\frac{\partial}{\partial \rho}\left(\rho^2\frac{\partial Z_1^{\text{out}}}{\partial \rho}\right)+\frac{\partial}{\partial \bar{\mu}}\left[(1-\bar{\mu}^2)\frac{\partial Z_1^{\text{out}}}{\partial \bar{\mu}}\right]\right\}$$
$$= \bar{\mu}\frac{\partial Z_1^{\text{out}}}{\partial \rho}+\frac{(1-\bar{\mu}^2)}{\rho}\frac{\partial Z_1^{\text{out}}}{\partial \bar{\mu}}$$
$$-\frac{4}{3}A_2Z_{00}\left\{\frac{e^{-\rho/2(1-\bar{\mu})}}{4\rho^2}\left[-2P_0(\bar{\mu})+\left(3+\frac{6}{\rho}\right)P_1(\bar{\mu})+P_2(\bar{\mu})\right]\right\} \quad (23)$$

which may then be rearranged to be

$$\frac{1}{\rho}\left\{\frac{\partial}{\partial \rho}\left(\rho^2\frac{\partial Z^*}{\partial \rho}\right)+\frac{\partial}{\partial \bar{\mu}}\left[(1-\bar{\mu}^2)\frac{\partial Z^*}{\partial \bar{\mu}}\right]\right\} = \frac{Z^*}{4}$$
$$+\frac{e^{-\rho/2}}{4\rho^2}\left[-2P_0(\bar{\mu})+\left(3+\frac{6}{\rho}\right)P_1(\bar{\mu})+P_2(\bar{\mu})\right] \quad (24)$$

via the substitution of the new dependent variable defined as

$$Z_1^{\text{out}}(r,\bar{\mu}) = -(\tfrac{4}{3}A_2Z_{00})\,e^{1/2\rho\bar{\mu}}Z^*(r,\bar{\mu}). \quad (25)$$

Since the equation is now non-homogeneous, its solution consists of two parts: the homogeneous solution and the particular solution. The homogeneous solution can be obtained by the separation of variables technique whereas the particular solution is a linear combination of the first three Legendre functions. The total solution is

$$Z_1^{\text{out}}(\rho,\bar{\mu}) = -A_2 Z_{00}\frac{4\,e^{1/2\rho\bar{\mu}}}{3}\left[\sqrt{\left(\frac{\pi}{\rho}\right)}\sum_{n=0}^{\infty} C_n K_{n+1/2}(\rho/2)P_n(\bar{\mu}) + \sum_{j=0}^{2} R_j(\rho)P_j(\bar{\mu})\right] \quad (26)$$

where the functions $R_j(\rho)$, $j = 0, 1, 2$ are

$$R_0(\rho) = \frac{e^{1/2\rho}}{2\rho}\int_\rho^\infty \frac{e^{-x}}{x}\mathrm{d}x + \frac{e^{-1/2\rho}}{2\rho}\ln\rho \quad (27a)$$

$$R_1(\rho) = \frac{3\,e^{1/2\rho}}{4\rho}\left(1-\frac{2}{\rho}\right)\int_\rho^\infty \frac{e^{-x}}{x}\mathrm{d}x - \frac{3\,e^{-1/2\rho}}{2\rho}\left[\left(\frac{1}{2}+\frac{1}{\rho}\right)\ln\rho + \frac{1}{\rho}\right] \quad (27b)$$

$$R_2(\rho) = \frac{e^{1/2\rho}}{4\rho}\left(1-\frac{6}{\rho}+\frac{12}{\rho^2}\right)\int_\rho^\infty \frac{e^{-x}}{x}\mathrm{d}x + \frac{3\,e^{-1/2\rho}}{4\rho}\left[\left(\frac{1}{3}+\frac{2}{\rho}+\frac{4}{\rho^2}\right)\ln\rho + \left(\frac{2}{\rho}+\frac{12}{\rho^2}\right)\right]. \quad (27c)$$

The constants C_n can be evaluated from the matching requirement as usual:

$$\lim_{r\to\infty}[Z_0^{\text{in}}(r,\bar{\mu})+\varepsilon Z_1^{\text{in}}(r,\bar{\mu})] = \lim_{\rho\to 0}[\varepsilon Z_0^{\text{out}}(\rho,\bar{\mu})+\varepsilon^2 Z_1^{\text{out}}(\rho,\bar{\mu})] \quad (28)$$

so that

$$C_0 = \frac{-3}{4\pi A_2}\left(\frac{Z_{01}}{Z_{00}}+\frac{1}{2}\right)+\frac{\gamma}{2\pi} \quad (29a)$$

$$C_1 = \frac{3(1-\gamma)}{4\pi} \quad (29b)$$

$$C_2 = \frac{(\gamma-3)}{4\pi} \quad (29c)$$

where γ is the Euler–Mascheroni constant with a value of 0.577215.

Since the values of the Legendre functions are identically zero except for the zero order when they are integrated over the drop surface, the electric field becomes obscure unless the analysis, at least the inner solution, is carried out one order further. To do that, we need to solve equation (18) by letting $n = 2$, and rearrange the RHS in terms of Legendre polynomials by using their orthogonality properties. With the algebra omitted, the results of the RHS and the solutions are given below:

$$\text{RHS}_2^{\text{in}} = P_0(\bar{\mu})\sum_{n=0}^{8}\frac{\omega_n^0}{r^{n+1}} + P_1(\bar{\mu})\sum_{n=0}^{6}\frac{\omega_n^1}{r^{n+2}} + P_2(\bar{\mu})\sum_{n=0}^{8}\frac{\omega_n^2}{r^{n+1}} + P_3(\bar{\mu})\sum_{n=0}^{5}\frac{\omega_n^3}{r^{n+3}} + P_4(\bar{\mu})\sum_{n=0}^{4}\frac{\omega_n^4}{r^{n+5}} \quad (30)$$

$$f_{02}(r) = E_0\left(1-\frac{1}{r}\right)+\frac{\omega_0^0}{2}\left(r-\frac{1}{r}\right)+\ln r\left(\omega_1^0-\frac{\omega_2^0}{r}\right) + \frac{\omega_3^0}{2}\left(\frac{1}{r^2}-\frac{1}{r}\right)+\frac{\omega_4^0}{6}\left(\frac{1}{r^3}-\frac{1}{r}\right)+\frac{\omega_5^0}{12}\left(\frac{1}{r^4}-\frac{1}{r}\right) + \frac{\omega_6^0}{20}\left(\frac{1}{r^5}-\frac{1}{r}\right)+\frac{\omega_7^0}{30}\left(\frac{1}{r^6}-\frac{1}{r}\right)+\frac{\omega_8^0}{42}\left(\frac{1}{r^7}-\frac{1}{r}\right) \quad (31a)$$

$$f_{12}(r) = E_1\left(r-\frac{1}{r^2}\right)-\frac{\omega_0^1}{2}\left(1-\frac{1}{r^2}\right)-\frac{\omega_1^1}{2}\left(\frac{1}{r}-\frac{1}{r^2}\right) - \frac{\omega_2^1}{3}\left(\frac{\ln r}{r^2}\right)+\frac{\omega_3^1}{4}\left(\frac{1}{r^3}-\frac{1}{r^2}\right)+\frac{\omega_4^1}{10}\left(\frac{1}{r^4}-\frac{1}{r^2}\right) + \frac{\omega_5^1}{18}\left(\frac{1}{r^5}-\frac{1}{r^2}\right)+\frac{\omega_6^1}{28}\left(\frac{1}{r^6}-\frac{1}{r^2}\right) \quad (31b)$$

$$f_{22}(r) = E_2\left(r^2-\frac{1}{r^3}\right)-\frac{\omega_0^2}{4}\left(r-\frac{1}{r^3}\right)-\frac{\omega_1^2}{6}\left(1-\frac{1}{r^3}\right) - \frac{\omega_2^2}{6}\left(\frac{1}{r}-\frac{1}{r^3}\right)-\frac{\omega_3^2}{4}\left(\frac{1}{r^2}-\frac{1}{r^3}\right) - \frac{\omega_4^2}{5}\left(\frac{\ln r}{r^3}\right)+\frac{\omega_5^2}{6}\left(\frac{1}{r^4}-\frac{1}{r^3}\right)+\frac{\omega_6^2}{14}\left(\frac{1}{r^5}-\frac{1}{r^3}\right) + \frac{\omega_7^2}{24}\left(\frac{1}{r^6}-\frac{1}{r^3}\right)+\frac{\omega_8^2}{36}\left(\frac{1}{r^7}-\frac{1}{r^3}\right) \quad (31c)$$

$$f_{32}(r) = E_3\left(r^3-\frac{1}{r^4}\right)-\frac{\omega_0^3}{12}\left(\frac{1}{r}-\frac{1}{r^4}\right) - \frac{\omega_1^3}{10}\left(\frac{1}{r^2}-\frac{1}{r^4}\right)-\frac{\omega_2^3}{6}\left(\frac{1}{r^3}-\frac{1}{r^4}\right)-\frac{\omega_3^3}{7}\left(\frac{\ln r}{r^4}\right) + \frac{\omega_4^3}{8}\left(\frac{1}{r^5}-\frac{1}{r^4}\right)+\frac{\omega_5^3}{18}\left(\frac{1}{r^6}-\frac{1}{r^4}\right) \quad (31d)$$

$$f_{42}(r) = E_4\left(r^4-\frac{1}{r^5}\right)-\frac{\omega_0^4}{14}\left(\frac{1}{r^3}-\frac{1}{r^5}\right) - \frac{\omega_1^4}{8}\left(\frac{1}{r^4}-\frac{1}{r^5}\right)-\frac{\omega_2^4}{9}\left(\frac{\ln r}{r^5}\right)+\frac{\omega_3^4}{10}\left(\frac{1}{r^6}-\frac{1}{r^5}\right) + \frac{\omega_4^4}{22}\left(\frac{1}{r^7}-\frac{1}{r^5}\right) \quad (31e)$$

where the ω_i^j values are known functions of the transport and electrical properties listed in Appendix A, and E_n are constants of integration remaining to be determined. As pointed out by Acrivos and Taylor [14], the second order inner solution contains a term that cannot be matched with the outer solution. Consequently, the inner solution should have a term $\varepsilon^2\ln\varepsilon Z_2^{\text{in},*}$ where $Z_2^{\text{in},*}$ is the solution of equation (10) with the interfacial constraint Z_{00} replaced by zero. That is

$$Z_2^{\text{in},*}(r,\bar{\mu}) = \omega_1^0\left(1-\frac{1}{r}\right). \tag{32}$$

With the matching condition

$$\lim_{r\to\infty}[Z_0^{\text{in}}+\varepsilon Z_1^{\text{in}}+\varepsilon^2\ln\varepsilon Z_2^{\text{in},*}+\varepsilon^2 Z_2^{\text{in}}] = \lim_{\rho\to 0}[\varepsilon Z_0^{\text{out}}+\varepsilon^2 Z_1^{\text{out}}] \tag{33}$$

the constant E_n values can be evaluated to be

$$E_0 = -\frac{4}{3}A_2 Z_{00}\left(\frac{3}{8}-\frac{\gamma}{2}\right)-\frac{1}{2}\left(\frac{Z_{00}}{2}+Z_{01}\right) \tag{34a}$$

$$E_1 = -\tfrac{1}{4}Z_{00} \tag{34b}$$

$$E_2 = E_3 = E_4 = 0. \tag{34c}$$

Further examination of equation (33) reveals that the next term of the outer expansion would be ε^3 ln $\varepsilon Z_2^{\text{out}}$ where Z_2^{out} can be shown to satisfy the equation identical to (12) and the matching requirement valued at $(-\omega_1^0/\rho)$ as $\rho\to 0$. It thus follows that

$$Z_2^{\text{out}}(\rho,\bar{\mu}) = -\frac{2}{3}A_2\left(\frac{Z_{00}}{\rho}\right)e^{-\rho/2(1-\bar{\mu})}. \tag{35}$$

As a consequence of the presence of equation (35) in the outer expansion, the next term of the inner solution must be $-2/3(A_2\varepsilon^3\ln\varepsilon Z_1^{\text{in}})$. From the arguments above, it is therefore concluded that the first few terms of the inner series expansion are expressed as

$$Z^{\text{in}}(r,\bar{\mu}) = Z_0^{\text{in}}(r,\bar{\mu})+\left(\varepsilon-\frac{2}{3}A_2\varepsilon^3\ln\varepsilon\right)Z_1^{\text{in}}(r,\bar{\mu}) +\varepsilon^2\ln\varepsilon\left(1-\frac{1}{r}\right)\omega_1^0+\varepsilon^2 Z_2^{\text{in}}(r,\bar{\mu})+\cdots \tag{36}$$

and the accuracy of the resulting expression may be demonstrated to be $O(\varepsilon^3)$, $O[\varepsilon^4(\ln\varepsilon)^2]$, and $O(\varepsilon^4\ln\varepsilon)$ respectively.

3.3. *Droplet heating/cooling*

Unlike the exterior problem, the classical perturbation yields uniform solution throughout the drop domain. It is therefore suitable to expand the drop temperature in a perturbation series as

$$\hat{Z}(r,\bar{\mu}) = \sum_{n=0}^{\infty}\varepsilon^n\hat{Z}_n(r,\bar{\mu}) \tag{37}$$

so that the lowest order of approximation to the governing equation can be obtained by neglecting the convective terms in equation (3). In view of the boundary condition associated with it, we conclude that this order is not dependent upon the polar coordinate. That is

$$\frac{\partial\hat{Z}_0}{\partial\tau} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\hat{Z}_0}{\partial r}\right) \tag{38}$$

which is the pure conduction for a spherical body. It is subjected to the initial temperature along with the continuity of temperature, and of heat flux at the interface as described by equations (5). In addition to those requirements, we impose the condition of zero heat flux at the drop center.

It is possible to solve equation (38) by the separation of variables technique together with the help of Duhamel's theorem to account for the time-dependent boundary condition. However, such an approach would lead to an integral equation of Voltera type characterizing the time evolution of surface temperature. An alternative approximation is the Method of Weighted Residuals (MWR) that may yield an approximation which often contains the main features of the result with only a few terms. In this method, the dependent quantity is expanded in a series of known functions $\hat{\mathscr{R}}_k(r)$ as

$$\hat{Z}_0(\tau,r) = Z_{00}(\tau)+\sum_{k=0}^{N_0}\hat{A}_k(\tau)\hat{\mathscr{R}}_k(r) \tag{39}$$

where $\hat{A}_k$ are unknown coefficients to be determined in a manner that the differential equation is satisfied in some best sense. Next we substitute the expansion (39) into the residual, $\hat{\mathscr{R}}$, defined as

$$\hat{\mathscr{R}} = \frac{\partial\hat{Z}_0}{\partial\tau}-\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\hat{Z}_0}{\partial r}\right). \tag{40}$$

It is clear that the further $\hat{Z}_0$ departs from the exact solution, the larger is the residual $\hat{\mathscr{R}}$. To control the growth of the residual, we choose the Galerkin method, which makes the residual orthogonal to the weighting function, thus making the residual approach zero as $N_0\to\infty$. This key feature relies upon a theorem which states that if a function is orthogonal to each member of a complete set of functions then that function can only be zero. By utilizing this theorem, the following algebraic/differential system can be derived for the determination of $\hat{A}_k$:

$$\frac{dZ_{00}}{d\tau}+\frac{4}{5}\frac{d\hat{A}_0}{d\tau}+\frac{4}{35}\frac{d\hat{A}_1}{d\tau}+\frac{4}{105}\frac{d\hat{A}_2}{d\tau} = -6\hat{A}_0+2\hat{A}_1+\frac{2}{5}\hat{A}_2 \tag{41a}$$

$$\frac{dZ_{00}}{d\tau}+\frac{4}{7}\frac{d\hat{A}_0}{d\tau}+\frac{4}{21}\frac{d\hat{A}_1}{d\tau}+\frac{20}{231}\frac{d\hat{A}_2}{d\tau} = -6\hat{A}_0-\frac{18}{7}\hat{A}_1-\frac{10}{7}\hat{A}_2 \tag{41b}$$

$$\frac{dZ_{00}}{d\tau}+\frac{4}{9}\frac{d\hat{A}_0}{d\tau}+\frac{20}{99}\frac{d\hat{A}_1}{d\tau}+\frac{140}{1287}\frac{d\hat{A}_2}{d\tau} = -6\hat{A}_0-\frac{322}{63}\hat{A}_1-\frac{370}{99}\hat{A}_2 \tag{41c}$$

$$Z_{00} = \frac{2}{\Phi_\kappa}(\hat{A}_0+\hat{A}_1+\hat{A}_2) \tag{41d}$$

where we have approximated the solution by a polynomial, rearranged in the form $(1-r^2)r^{2k}$ so that some of the boundary conditions are satisfied automatically for the trial functions $\hat{\mathscr{R}}_k(r)$ because of their simplicity. The fourth equation, obtained directly from the interfacial energy balance, relates the leading order surface temperature to the coefficients $\hat{A}_k$.

The initial conditions may be obtained by weighting the initial residual with the trial functions in conjunction with the requirement regarding the continuity of heat flux at the interface. Thus

$$\hat{A}_0(0) = \frac{69\Phi_\kappa}{32(27+2\Phi_\kappa)},$$

$$\hat{A}_1(0) = \frac{-33\Phi_\kappa}{16(27+2\Phi_\kappa)},$$

$$\hat{A}_2(0) = \frac{429\Phi_\kappa}{32(27+2\Phi_\kappa)}. \quad (41e\text{–}g)$$

By eliminating the variables, one can convert the system (41) to a single third order homogeneous ODE with constant coefficients, for which a standard treatment is readily available. That is, the solution is a linear combination of three exponential terms

$$\hat{A}_0(\tau) = \beta_{1,00}\,e^{-a_0\tau}+\beta_{2,00}\,e^{-a_1\tau}+\beta_{3,00}\,e^{-a_2\tau} \quad (42a)$$

$$\hat{A}_2(\tau) = \beta_{4,00}\,e^{-a_0\tau}+\beta_{5,00}\,e^{-a_1\tau}+\beta_{6,00}\,e^{-a_2\tau} \quad (42b)$$

$$\hat{A}_3(\tau) = \beta_{7,00}\,e^{-a_0\tau}+\beta_{8,00}\,e^{-a_1\tau}+\beta_{9,00}\,e^{-a_2\tau} \quad (42c)$$

where a_k are the roots of the associated characteristic equation

$$(4\Phi_\kappa+54)x^3+(558\Phi_\kappa+3267)x^2 + (18\,810\Phi_\kappa+45\,045)x+135\,135 = 0, \quad (42d)$$

and the integrating constants are evaluated from expressions (41e–g) along with the remaining initial conditions derived from equations (41). These are

$$\left.\frac{d^{i+1}\hat{A}_2}{d\tau^{i+1}}\right|_{\tau=0} = \frac{-3003}{3024+224\Phi_\kappa}\left\{(64+11\Phi_\kappa)\frac{d^i\hat{A}_2}{d\tau^i} + (20+7\Phi_\kappa)\frac{d^i\hat{A}_1}{d\tau^i}+3\Phi_\kappa\frac{d^i\hat{A}_0}{d\tau^i}\right\}\right|_{\tau=0} \quad (42e)$$

$$\left.\frac{d^{i+1}\hat{A}_1}{d\tau^{i+1}}\right|_{\tau=0} = -\frac{2}{13}\left.\frac{d^{i+1}\hat{A}_2}{d\tau^{i+1}}\right|_{\tau=0}+42\left.\frac{d^i\hat{A}_2}{d\tau^i}\right|_{\tau=0} \quad (42f)$$

$$\left.\frac{d^{i+1}\hat{A}_0}{d\tau^{i+1}}\right|_{\tau=0} = \frac{23}{143}\left.\frac{d^{i+1}\hat{A}_2}{d\tau^{i+1}}\right|_{\tau=0} +22\left.\frac{d^i\hat{A}_2}{d\tau^i}\right|_{\tau=0}+20\left.\frac{d^i\hat{A}_1}{d\tau^i}\right|_{\tau=0} \quad (42g)$$

where the superscripts represent the order of differentiation. The above six equations, three for each value of i ($i = 0, 1$) plus those stated earlier, constitute a system of nine equations in nine unknown β values.

At higher orders, the equation reduces to the standard form of axisymmetric heat conduction problem in spherical coordinates with the convective term, known from the preceding approximation, considered as a heat source. In general the equation for any order may recursively be given by

$$\frac{\partial\hat{Z}_n}{\partial\tau} = \frac{1}{r^2}\left\{\frac{\partial}{\partial r}\left(r^2\frac{\partial\hat{Z}_n}{\partial r}\right)+\frac{\partial}{\partial\bar{\mu}}\left[(1-\bar{\mu}^2)\frac{\partial\hat{Z}_n}{\partial\bar{\mu}}\right]\right\} - \frac{1}{\Phi_\alpha}\left[\hat{U}_r\frac{\partial\hat{Z}_{n-1}}{\partial r}-\hat{U}_\theta\frac{(1-\bar{\mu}^2)^{1/2}}{r}\frac{\partial\hat{Z}_{n-1}}{\partial\bar{\mu}}\right]. \quad (43)$$

Due to the non-angular dependency of the lowest order solution, the first order approximation, $n = 1$, is governed by equation (43) with the angular convective term identically equal to zero. Perhaps the best approach of solution would be to express the radial convective contribution, denoted by $\widehat{\mathrm{RCC}}_{k1}$, in terms of Legendre functions as follows

$$\widehat{\mathrm{RCC}}_{01} = 0 \quad (44a)$$

$$\widehat{\mathrm{RCC}}_{11} = -\frac{\Phi_\mu(1-r^2)}{\Phi_\alpha(1+\Phi_\mu)}\{[r\beta_{1,00}-(r-2r^3)\beta_{4,00} -(2r^3-3r^5)\beta_{7,00}]\,e^{-a_0\tau} +[r\beta_{2,00}-(r-2r^3)\beta_{5,00}-(2r^3-3r^5)\beta_{8,00}]\,e^{-a_1\tau} +[r\beta_{3,00}-(r-2r^3)\beta_{6,00}-(2r^3-3r^5)\beta_{9,00}]\,e^{-a_2\tau}\} \quad (44b)$$

$$\widehat{\mathrm{RCC}}_{21} = \frac{W(1-r^2)}{\Phi_\alpha(1+\Phi_\mu)}\{[r^2\beta_{1,00}-(r^2-2r^4)\beta_{4,00} -(2r^4-3r^6)\beta_{7,00}]\,e^{-a_0\tau} +[r^2\beta_{2,00}-(r^2-2r^4)\beta_{5,00}-(2r^4-3r^6)\beta_{8,00}]\,e^{-a_1\tau} +[r^2\beta_{3,00}-(r^2-2r^4)\beta_{6,00}-(2r^4-3r^6)\beta_{9,00}]\,e^{-a_2\tau}\}. \quad (44c)$$

In order to meet the consistency of the interfacial temperature, and to satisfy the finite value requirement at the origin, the solution is sought in the form

$$\hat{Z}_1(r,\bar{\mu}) = \sum_{k=0}^{2}\hat{f}_{k1}(r)P_k(\bar{\mu}) \quad (45)$$

so that equation (43) can be solved termwise. The advantage of being termwise is the relative ease with regard to the solution due to the reduction of the number of variables by one. The analysis of the resulting equations is very much the same as that of the leading order even with the non-homogeneity introduced by the heat source term.

Since the steps leading to the solution were discussed earlier, we need not repeat them here. Instead, we present the solution and refer the reader elsewhere [15] for details

$$\hat{f}_{01}(\tau,r) = Z_{01}(\tau)+(1-r^2)\sum_{k=0}^{N_{01}}r^{2k}\hat{B}_k(\tau) \quad (46a)$$

$$\hat{f}_{11}(\tau, r) = rZ_{11}(\tau) + (1 - r^2) \sum_{k=0}^{N_{11}} r^{2k+1} \hat{C}_k(\tau) \quad (46b)$$

$$\hat{f}_{21}(\tau, r) = r^2 Z_{21}(\tau) + (1 - r^2) \sum_{k=0}^{N_{21}} r^{2k+2} \hat{D}_k(\tau) \quad (46c)$$

in which we have used trial functions that are nearly the same as the terms of their corresponding $\widehat{\mathrm{RCC}}_{k1}$ values. The presence of the extra terms is to satisfy part of the problem. Since the function f_{k1} values are generally weak [15], we retain only the first two terms of the series so that the first order interfacial temperature becomes

$$Z_{01}(\tau) = \frac{2}{\Phi_\kappa}[\hat{B}_0(\tau) + \hat{B}_1(\tau)] - \frac{1}{2} Z_{00}(\tau) \quad (46d)$$

$$Z_{11}(\tau) = \frac{1}{1 + 2\Phi_\kappa}\left[2\hat{C}_0(\tau) + 2\hat{C}_1(\tau) + \frac{\Phi_\kappa}{8}\left(\frac{3 + 4\Phi_\mu}{1 + \Phi_\mu}\right) Z_{00}(\tau)\right] \quad (46e)$$

$$Z_{21}(\tau) = \frac{1}{2 + 3\Phi_\kappa}\left[2\hat{D}_0(\tau) + 2\hat{D}_1(\tau) + \frac{\Phi_\kappa}{24}\left(\frac{W}{1 + \Phi_\mu}\right) Z_{00}(\tau)\right] \quad (46f)$$

where the constants $\hat{B}_i$, $\hat{C}_i$, and $\hat{D}_i$ are given in Appendix B.

From a study of Nguyen and Chung [16], the temperature inside a vaporizing drop translating in an electric field is almost spherically symmetric, and the first order interfacial temperature is, in any circumstances, more than one order-of-magnitude smaller than the leading order. This indicates that higher order analysis is practically unnecessary.

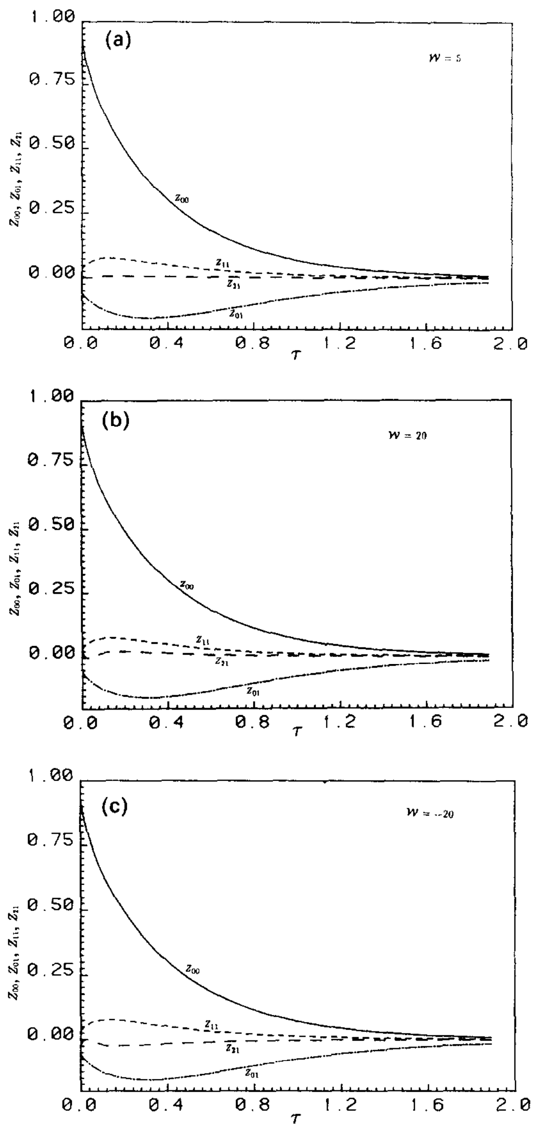

FIG. 1. Time evolution of surface temperature for $W = 5$ (a), 20 (b) and -20 (c).

4. RESULTS AND DISCUSSION

In this section we intend to carry out a parametric study, rather than being referenced to a particular type of material, of the solutions obtained in the previous section in order to demonstrate the electric field/two-phase flow interactions and the consequence of their influences upon the heat transfer process. Efforts will also be made to demonstrate the usefulness of the present treatment over a more direct method that usually involves the solution of the so-called Volterra integral equation. Based on the authors' knowledge, this work is the first that employs a method of weighted residuals to a boundary value problem involving time-dependent boundary conditions and/or non-homogeneities arising within the governing equation itself.

Perhaps the most important parameters in a conjugate system, such as a two-phase system, are the interfacial variables which refer to the surface temperature in this case. This is true because once it is known, their couplings are eliminated, and one is allowed to model the transfer processes for each region separately. In the study of Nguyen [15] and a related one Nguyen and Chung [16], the most dominant surface temperature is that associated with the zero order solution. It is, therefore, necessary to retain as many terms as possible in order to assure that the solution, though somewhat approximate, preserves the main features of the important mechanisms occurring during the course of exposure. Figures 1(a)–(c) show the transient response of various components of temperature for a system composed of two immiscible fluids having similar physical properties at three different values of W (5, 20, and -20). For a positive W, the electrically generated flow, directed from the pole to the equator, causes a convective effect where more than 77% of the surface area senses an increase in temperature. Although the influence of the electric

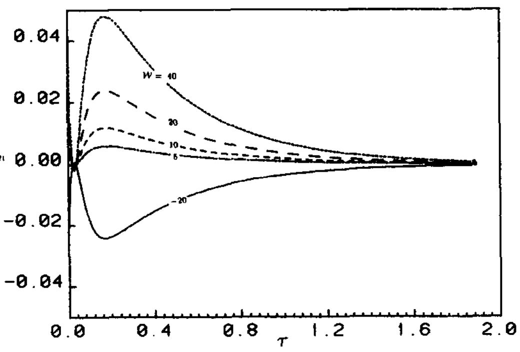

FIG. 2. The electric field effect on the surface temperature.

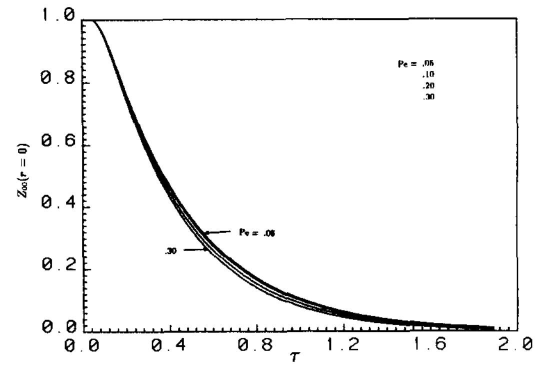

FIG. 3. Time variation of the temperature at the center of the droplet.

field has very little, if any, impact on the history of the surface temperature, its presence is clearly shown to delay the establishment of a local thermal equilibrium at the interface. As will be discussed shortly, the influence of the electric field corresponding to this order does not yield any net effect, but it restructures the temperature distribution on the drop surface, and hence the heat source distributed inside the drop domain for the next higher order approximation. For a negative W, the electric field-driven flow reverses its direction causing effects that are totally opposite to the former situation, but its overall result stays unchanged. One should not be misled that the direction of the internal circulation is a controlling parameter; it is instead dictated by the electrical properties of the participating medium. One distinctive characteristic of the leading and first order solutions is the fast response with time for the former, especially at short time, whereas the latter tends to establish its maximum influence, but can never overcome the domination of the conduction.

Unlike the case of thermal transport at high Peclet numbers where the electric field has a decisive role even with a relatively low value of W, no pronounced consequences are observed in this study for a dimensionless field strength as high as 5. This may be explained by the fact that conduction is the superior mode of heat transport at low Peclet numbers. However, as the field is increased in potential, the contribution from the electrically induced convection becomes more competitive over its counterpart, and it eventually outplays that due to translation provided that the applied voltage is sufficient. This behavior is clearly demonstrated in Fig. 1(b), where the influence of the field becomes noticeable. In carrying out the computations, we have used three and two terms for the zero and first order approximations respectively. Since the MWR method is very similar to the method of separation of variables, they both suffer the same difficulties in getting the solution to converge at small time. Based on this information and the monotonic nature of the solution, the maximum error occurs at $\tau = 0$, which is about 7% lower than the exact prescribed initial value.

In Fig. 2 we illustrate the effects of an electric field on the interfacial temperature, Z_{21}, at various field strengths. They all show a general trend with the early stage associated with a rapid increase in temperature until the electric field effect is fully established when it reaches a critical value, then falls off, at a slower pace however, as time increases. The time it takes to achieve a maximum/minimum value is roughly the same for all cases given in the figure, and their maximum values exhibit a linear relationship with W. It is noted that the initial condition was not satisfied exactly by the MWR method, but the error is negligible, as shown in Fig. 2.

The fully established thermal equilibrium can be characterized by the temperature of the drop center because a homogeneous material can be considered to be in thermal equilibrium when all the temperature gradients have vanished. In Fig. 3 we plot the temperature at the drop center against time for different values of Peclet numbers. In general, the temperature at the location falls off in an exponential decay manner. Although it behaves as if it is shielded from the field, one should be reminded that it does have some effects if a higher order approximation is included in the analysis. It is important to note that the length of the transient period is a weak function of the Peclet number, and this fact may be used to substantiate our assumption that there is no region within the drop where convection and conduction are of the same order. On this basis, the application of regular perturbation is justified.

As discussed earlier, once the interfacial temperature is known the transport processes within their own phases may be modeled individually. With the help of the inner solution of the continuous phase, the local heat flux along the periphery of the drop can be calculated by evaluating the normal derivative at $r = 1$. This operation results in

$$Q''(\tau, \bar{\mu}) = \left\{ Z_{00} + \left(\varepsilon - \frac{2}{3} A_2 \varepsilon^3 \ln \varepsilon \right) \left(\frac{Z_{00}}{2} + Z_{01} \right) \right.$$

$$+ \frac{1}{6} \varepsilon^2 \ln \varepsilon \left(\frac{3 + 2\Phi_\mu}{1 + \Phi_\mu} \right) Z_{00} - \varepsilon^2 \left[E_0 + \omega_0^0 \right.$$

$$+\omega_0^1-\sum_{k=2}^{8}\frac{\omega_k^0}{k-1}\Bigg]\Bigg\}P_0(\bar{\mu})$$

$$+\left\{\left(\varepsilon-\frac{2}{3}A_2\varepsilon^3\ln\varepsilon\right)\left[2Z_{11}-\frac{3+4\Phi_\mu}{8(1+\Phi_\mu)}Z_{00}\right]\right.$$

$$\left.+\varepsilon^2\left[\frac{3}{4}Z_{00}+\sum_{k=0}^{6}\frac{\omega_k^1}{k+1}\right]\right\}P_1(\bar{\mu})$$

$$+\left\{\left(\varepsilon-\frac{2}{3}A_2\varepsilon^3\ln\varepsilon\right)\left[3Z_{21}-\frac{W}{24(1+\Phi_\mu)}Z_{00}\right]\right.$$

$$\left.+\varepsilon^2\sum_{k=0}^{8}\frac{\omega_k^2}{k+1}\right\}P_2(\bar{\mu})+\varepsilon^2\left\{\sum_{k=0}^{5}\frac{\omega_k^3}{k+4}\right\}P_3(\bar{\mu})$$

$$+\varepsilon^2\left\{\sum_{k=0}^{4}\frac{\omega_k^4}{k+7}\right\}P_4(\bar{\mu}) \tag{47}$$

where the heat flux has been non-dimensionalized by $\kappa(T_0-T_\infty)/R$. Also of interest is the total heat transfer, Q, which can be obtained by integrating the heat flux over the entire drop surface. Such integration results in no net contributions from terms with Legendre polynomials of orders other than zero. That is

$$Q(\tau)=2\left\{Z_{00}+\left(\varepsilon-\frac{2}{3}A_2\varepsilon^3\ln\varepsilon\right)\left(\frac{Z_{00}}{2}+Z_{01}\right)\right.$$

$$+\frac{1}{6}\varepsilon^2\ln\varepsilon\left(\frac{3+2\Phi_\mu}{1+\Phi_\mu}\right)Z_{00}$$

$$\left.-\varepsilon^2\left[E_0+\omega_0^0+\omega_1^0-\sum_{k=2}^{8}\frac{\omega_k^0}{k-1}\right]\right\}. \tag{48}$$

From the properties of Legendre functions, it appears that only ω_k^0, $k=4$–8, depends, among other transport properties, on the electric field. This indicates that only the zero and first order terms carry the contribution due to the electric field. Therefore, one may interpret the first term as the contribution due to conduction, the second and third to be the convective enhancement due to translational motion of the drop, and the fourth term to represent the combined convection of the former and the electro-convection modes. It should be noted that the functional form of equation (48) is different from the analysis of Griffiths and Morrison [7] where they derived the Nusselt number to be a series with terms of even power in Peclet number for a stationary drop subjected to an electric field. Such information, if known in advance, would be very helpful in correlating experimental data. It is also worthwhile to point out that the present result reduces to the expression given by Acrivos and Taylor [14] when $Z_{00}=1$ and $Z_{k1}=0$ for a slowly translating solid sphere in an electric field-free environment

$$Q_{\text{solid}}=2+\varepsilon+\varepsilon^2\left(\gamma+\frac{241}{480}+\ln\varepsilon+\frac{1}{2}\varepsilon\ln\varepsilon\right). \tag{49}$$

To assess the advantage of the use of an electric field to enhance heat transfer, it is desirable to examine the ratio of the total transport rate to that without an electric field. Here we give the results at two different Peclet numbers in Fig. 4. It is seen that the curves do not collapse into one at small time as boundary layer theory had predicted. This result indicates that convection, especially electro-convection, has a major role at the later stage of the process. It also reveals that the net effect due to the electric field depends strongly on the translation, the magnitude of which can be deduced from equation (48) to be $\varepsilon^2 W$.

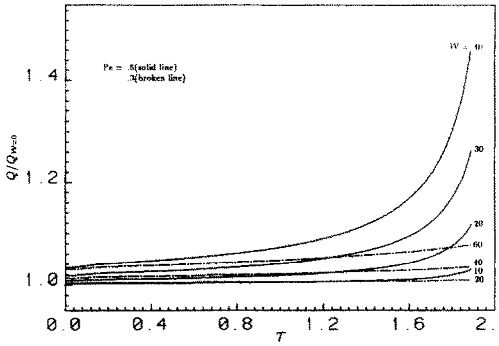

FIG. 4. The effect of an electric field on the heat transfer rate.

5. CONCLUDING REMARKS

The results illustrated thus far would fill the lower end of the Peclet number spectrum that has not been explored in the past. A number of interesting classical results can be deduced from this study by setting the parameters to their appropriate values. Although the electric field does stimulate the transport process to some extent, its usage is only effective at high values of W. None the less, one should be aware of the nature of electrohydrodynamic couplings as it may carry a direct application in combustion where a change in temperature distribution inside a heterogeneous droplet would enhance the likelihood of a secondary atomization. There is no doubt that such technology, if well developed, may lead to clean and more efficient combustion of fuel drops. The solution technique proves to be useful for the study of boundary value problems with a time varying interfacial condition that is one of the main attractions of our study. This feature is shown to be important for transient analysis, and should therefore be incorporated into the model.

REFERENCES

1. T. C. Scott, Surface area generation and droplet size control using pulsed electric fields, *A.I.Ch.E. Jl* **33,** 1557–1559 (1987).
2. N. Kaji, Y. H. Mori, Y. Tochitani and K. Komotori, Augmentation of direct-contact heat transfer to drops with an intermittent electric field, *J. Heat Transfer* **102,** 32–37 (1980).
3. N. Kaji, Y. H. Mori and Y. Tochitani, Electrically induced shape oscillation of drops as a means of direct-contact heat transfer enhancement: Part 2—Heat transfer, *J. Heat Transfer* **110,** 700–704 (1988).
4. R. Kronig and G. Ahsmann, The influence of an electric

field on the convective heat transfer in liquids, *Appl. Scient. Res. Sect.* **A2,** 31–32 (1949).
5. T. B. Jones, Electrohydrodynamically enhanced heat transfer in liquids—a review, *Adv. Heat Transfer* **14,** 107–148 (1978).
6. F. A. Morrison, Transient heat and mass transfer to a drop in an electric field, *J. Heat Transfer* **99,** 269–273 (1977).
7. S. K. Griffiths and F. A. Morrison, Low Peclet number heat and mass transfer from a drop in an electric field, *J. Heat Transfer* **101,** 484–488 (1979).
8. S. K. Griffiths and F. A. Morrison, The transport from a drop in an alternating electric field, *Int. J. Heat Mass Transfer* **26,** 717–726 (1983).
9. L. Sharpe and F. A. Morrison, Numerical analysis of heat and mass transfer from fluid spheres in an electric field, *J. Heat Transfer* **108,** 337–342 (1986).
10. J. N. Chung and D. L. R. Oliver, Transient heat transfer in a fluid sphere translating in an electric field, *J. Heat Transfer* **112,** 84–91 (1990).
11. B. Abramzon and I. Borde, Conjugate unsteady heat transfer from a droplet in creeping flow, *A.I.Ch.E. Jl* **26,** 536–544 (1980).
12. L. S. Chang, T. E. Carleson and J. C. Berg, Heat and mass transfer to a translating drop in an electric field, *Int. J. Heat Mass Transfer* **25,** 1023–1030 (1982).
13. L. S. Chang and J. C. Berg, Fluid flow and transfer behavior of a drop translating in an electric field at intermediate Reynolds numbers, *Int. J. Heat Mass Transfer* **26,** 823–832 (1983).
14. A. Acrivos and T. D. Taylor, Heat and mass transfer from single sphere in Stokes flow, *Physics Fluids* **5,** 387–394 (1962).
15. H. D. Nguyen, Studies of transport processes in an electric field, Ph.D. dissertation, Washington State University (1989).
16. H. D. Nguyen and J. N. Chung, Evaporation from a translating drop in an electric field, *Int. J. Heat Mass Transfer* (submitted for publication).

APPENDIX A

The following are the expressions for ω_i^j associated with equation (31) in the text:

$$\omega_0^0 = \frac{Z_{00}}{3} \tag{A1}$$

$$\omega_1^0 = \frac{2}{3}A_2Z_{00} \tag{A2}$$

$$\omega_2^0 = \frac{Z_{00}}{3} \tag{A3}$$

$$\omega_3^0 = -\frac{Z_{00}}{6}(A_1-6A_2+2A_1A_2-4A_2^2)-\frac{2}{3}A_2Z_{11} \tag{A4}$$

$$\omega_4^0 = -Z_{00}\left(\frac{2}{3}A_1A_2+\frac{2}{5}A_3^2\right) \tag{A5}$$

$$\omega_5^0 = Z_{00}\left(\frac{5}{3}A_1+2A_1A_2-A_1^2+A_3^2\right)-2A_1Z_{11}-\frac{6}{5}A_3Z_{21} \tag{A6}$$

$$\omega_6^0 = Z_{00}\left(\frac{4}{3}A_1^2+\frac{4}{15}A_3^2\right) \tag{A7}$$

$$\omega_7^0 = -\frac{5}{3}A_3^2Z_{00}+2A_3Z_{21} \tag{A8}$$

$$\omega_8^0 = \frac{4}{5}A_3^2Z_{00} \tag{A9}$$

$$\omega_0^1 = -\left(\frac{Z_{00}}{2}+Z_{01}\right) \tag{A10}$$

$$\omega_1^1 = -Z_{00}\left(A_2-\frac{1}{5}A_3\right)-2A_2Z_{01} \tag{A11}$$

$$\omega_2^1 = -\frac{Z_{00}}{5}(2A_3+3A_2A_3) \tag{A12}$$

$$\omega_3^1 = -Z_{00}\left(A_1-\frac{4}{15}A_3-\frac{2}{5}A_1A_3-\frac{9}{5}A_2A_3\right)-2A_1Z_{01}-\frac{6}{5}A_2Z_{21} \tag{A13}$$

$$\omega_4^1 = \frac{Z_{00}}{5}\left(2A_3-12A_1A_3+\frac{8}{3}A_2A_3\right) \tag{A14}$$

$$\omega_5^1 = -\frac{Z_{00}}{5}(2A_3-17A_1A_3+8A_2A_3)-\frac{8}{5}A_3Z_{11}-\frac{18}{5}A_1Z_{21} \tag{A15}$$

$$\omega_6^1 = -\frac{19}{15}A_1A_3Z_{00} \tag{A16}$$

$$\omega_0^2 = -\frac{Z_{00}}{3} \tag{A17}$$

$$\omega_1^2 = -\frac{5}{3}A_2Z_{00} \tag{A18}$$

$$\omega_2^2 = -Z_{00}(1+A_1+2A_2-2A_2^2)-2Z_{11} \tag{A19}$$

$$\omega_3^2 = Z_{00}\left(\frac{2}{3}A_1+\frac{5}{3}A_2-A_3-\frac{5}{3}A_1A_2+\frac{10}{3}A_2^2\right)-2A_3Z_{01}-\frac{10}{3}A_2Z_{11} \tag{A20}$$

$$\omega_4^2 = Z_{00}\left(\frac{5}{3}A_1A_2-\frac{4}{7}A_3^2\right) \tag{A21}$$

$$\omega_5^2 = Z_{00}\left(\frac{5}{3}A_1+\frac{5}{3}A_2-A_3-\frac{5}{3}A_1A_2+\frac{10}{3}A_2^2\right)-2A_3Z_{01}-\frac{10}{3}A_2Z_{11} \tag{A22}$$

$$\omega_6^2 = Z_{00}\left(\frac{5}{3}A_1^2+\frac{2}{21}A_3^2\right) \tag{A23}$$

$$\omega_7^2 = -\frac{40}{21}Z_{00}A_3^2+\frac{16}{7}A_3Z_{21} \tag{A24}$$

$$\omega_8^2 = \frac{20}{21}A_3^2Z_{00} \tag{A25}$$

$$\omega_0^3 = -\frac{6}{5}A_3Z_{00}-\frac{6}{5}Z_{21} \tag{A26}$$

$$\omega_1^3 = Z_{00}\left(\frac{5}{2}A_3-3A_2A_3\right)-\frac{9}{5}Z_{21} \tag{A27}$$

$$\omega_2^3 = \frac{Z_{00}}{5}(2A_3-6A_1A_3+32A_2A_3)-\frac{12}{5}A_3Z_{11}-\frac{24}{5}A_2Z_{21} \tag{A28}$$

$$\omega_3^3 = -\frac{Z_{00}}{5}(2A_3+6A_1A_3+4A_2A_3)-\frac{4}{5}A_3Z_{01} \tag{A29}$$

$$\omega_4^3 = -\frac{Z_{00}}{5}(6A_3 - 16A_1A_3 + 12A_2A_3) + \frac{12}{5}A_3Z_{11} - \frac{12}{5}A_1Z_{21} \quad \text{(A30)}$$

$$\omega_5^3 = -3A_1A_3Z_{00} \quad \text{(A31)}$$

$$\omega_0^4 = -\frac{36}{35}A_3^2Z_{00} \quad \text{(A32)}$$

$$\omega_1^4 = \frac{18}{7}A_3^2Z_{00} - \frac{108}{35}A_3Z_{21} \quad \text{(A33)}$$

$$\omega_2^4 = -\frac{36}{35}A_3^2Z_{00} \quad \text{(A34)}$$

$$\omega_3^4 = -\frac{10}{7}A_3^2Z_{00} + \frac{12}{7}A_3Z_{21} \quad \text{(A35)}$$

$$\omega_4^4 = \frac{32}{35}A_3^2Z_{00}. \quad \text{(A36)}$$

APPENDIX B

$$\hat{B}_0(\tau) = \beta_{0,01}\,e^{-a_0\tau} + \beta_{1,01}\,e^{-a_1\tau} + \beta_{2,01}\,e^{-a_2\tau} + \beta_{3,01}\,e^{-b_0\tau} + \beta_{4,01}\,e^{-b_1\tau} \quad \text{(B1)}$$

$$\hat{B}_1(\tau) = \beta_{5,01}\,e^{-a_0\tau} + \beta_{6,01}\,e^{-a_1\tau} + \beta_{7,01}\,e^{-a_2\tau} + \beta_{8,01}\,e^{-b_0\tau} + \beta_{9,01}\,e^{-b_1\tau} \quad \text{(B2)}$$

with the constants $\beta_{k,01}$ defined as

$$\beta_{k,01} = \left(\frac{7}{8}a_k^2 - \frac{105}{2}a_k\right)\left[\frac{\beta_{k+1,00} + \beta_{k+4,00} + \beta_{k+7,00}}{(7+\Phi_\kappa)a_k^2 - (105+42\Phi_\kappa)a_k + 315\Phi_\kappa}\right], \quad k = 0,\,1,\,2$$

$$\beta_{3,01} = \frac{b_1\gamma_1 + \gamma_2}{b_1 - b_0}, \quad \beta_{4,01} = \frac{b_0\gamma_1 + \gamma_2}{b_0 - b_1}$$

$$\beta_{k+5,01} = \frac{21\Phi_\kappa - 4(7+\Phi_\kappa)a_k}{420 + 31\Phi_\kappa}\beta_{k,01} + 7a_k\frac{\beta_{k+1,00} + \beta_{k+4,00} + \beta_{k+7,00}}{2(240 + 31\Phi_\kappa)}, \quad k = 0, 1, 2$$

$$\beta_{8,01} = \frac{21\Phi_\kappa - 4(7+\Phi_\kappa)b_0}{420 + 31\Phi_\kappa}\beta_{3,01}$$

$$\beta_{9,01} = \frac{21\Phi_\kappa - 4(7+\Phi_\kappa)b_1}{420 + 31\Phi_\kappa}\beta_{4,01}$$

where

$$\gamma_1 = \frac{189}{16(7+\Phi_\kappa)(27+2\Phi_\kappa)} - \beta_{0,01} - \beta_{1,01} - \beta_{2,01}$$

$$\gamma_2 = \frac{1701(35+2\Phi_\kappa)}{16(27+2\Phi_\kappa)(7+\Phi_\kappa)^2} + \sum_{k=0}^{2} a_k\left[\beta_{k,01} - 7\frac{\beta_{k+1,00} + \beta_{k+4,00} + \beta_{k+7,00}}{8(7+\Phi_\kappa)}\right]$$

$$b_0 = \frac{(105+42\Phi_\kappa) + \sqrt{(504\Phi_\kappa^2 + 11\,025)}}{2(7+\Phi_\kappa)}$$

$$b_1 = \frac{(105+42\Phi_\kappa) - \sqrt{(504\Phi_\kappa^2 + 11\,025)}}{2(7+\Phi_\kappa)}.$$

$$\hat{C}_0(\tau) = \beta_{5,11}\,e^{-a_0\tau} + \beta_{6,11}\,e^{-a_1\tau} + \beta_{7,11}\,e^{-a_2\tau} + \beta_{8,11}\,e^{-c_0\tau} + \beta_{9,11}\,e^{-c_1\tau} \quad \text{(B3)}$$

$$\hat{C}_1(\tau) = \beta_{0,11}\,e^{-a_0\tau} + \beta_{1,11}\,e^{-a_1\tau} + \beta_{2,11}\,e^{-a_2\tau} + \beta_{3,11}\,e^{-c_0\tau} + \beta_{4,11}\,e^{-c_1\tau} \quad \text{(B4)}$$

with the constants $\beta_{k,11}$ defined as

$$\beta_{k,11} = \frac{-\Phi_\mu\Delta_{k,11}}{2\Phi_\alpha(1+\Phi_\mu)}\Bigg\{7[a_k(11+4\Phi_\kappa) - 45(1+2\Phi_\kappa)][33\beta_{k+1,00} - 11\beta_{k+4,00} - 7\beta_{k+7,00}] - 21[a_k(9+4\Phi_\kappa) - 35(1+2\Phi_\kappa)]\left[11\beta_{k+1,00} - \beta_{k+4,00} - \frac{25}{13}\beta_{k+7,00}\right]\Bigg\} - \frac{231a_k^2}{8}\Delta_{k,11}\left(\frac{3+4\Phi_\mu}{1+\Phi_\mu}\right)[\beta_{k+1,00} + \beta_{k+4,00} + \beta_{k+7,00}], \quad k = 0, 1, 2$$

$$\beta_{3,11} = \frac{c_1\gamma_3 + \gamma_4}{c_1 - c_0}, \quad \beta_{4,11} = \frac{c_0\gamma_3 + \gamma_4}{c_0 - c_1}$$

$$\beta_{k+5,11} = \left[\frac{a_k(40+8\Phi_\kappa) - 33(37+18\Phi_\kappa)}{165(1+2\Phi_\kappa)}\right]\beta_{k,11} + \left[\frac{\Phi_\mu}{30\Phi_\alpha(1+\Phi_\mu)}\right]\Bigg\{\frac{11+4\Phi_\kappa}{1+2\Phi_\kappa}\left[3\beta_{k+1,00} - \beta_{k+4,00} - \frac{7}{11}\beta_{k+7,00}\right] - \frac{27+12\Phi_\kappa}{1+2\Phi_\kappa}\left[\beta_{k+1,00} - \frac{1}{11}\beta_{k+4,00} - \frac{25}{143}\beta_{k+7,00}\right]\Bigg\} + \frac{3+4\Phi_\mu}{1+\Phi_\mu}\left[\frac{\beta_{k+1,00} + \beta_{k+4,00} + \beta_{k+7,00}}{40(1+2\Phi_\kappa)}\right]a_k, \quad k = 0, 1, 2$$

$$\beta_{8,11} = \left[\frac{c_0(40+8\Phi_\kappa) - 33(37+18\Phi_\kappa)}{165(1+2\Phi_\kappa)}\right]\beta_{3,11}$$

$$\beta_{9,11} = \left[\frac{c_1(40+8\Phi_\kappa) - 33(37+18\Phi_\kappa)}{165(1+2\Phi_\kappa)}\right]\beta_{4,11}$$

where

$$\frac{1}{\Delta_{k,11}} = 28[(10+2\Phi_\kappa)a_k^2 - (309+156\Phi_\kappa)a_k + 1155(1+2\Phi_\kappa)], \quad k = 0, 1, 2$$

$$\gamma_3 = -\frac{3+4\Phi_\mu}{1+\Phi_\mu}\left[\frac{891\Phi_\kappa}{128(5+\Phi_\kappa)(27+2\Phi_\kappa)}\right] - \beta_{0,11} - \beta_{1,11} - \beta_{2,11}$$

$$\gamma_4 = \frac{2673}{256}\left(\frac{3+4\Phi_\mu}{1+\Phi_\mu}\right)\left[\frac{\Phi_\kappa(103+52\Phi_\kappa)}{(27+2\Phi_\kappa)(5+\Phi_\kappa)^2}\right] + \frac{11}{7(40+8\Phi_\kappa)}\Bigg\{\frac{\Phi_\mu}{\Phi_\alpha(1+\Phi_\mu)}\Bigg[(11+4\Phi_\kappa)\sum_{k=0}^{2}\left(\frac{21}{2}\beta_{k+1,00} - \frac{7}{2}\beta_{k+4,00} - \frac{49}{22}\beta_{k+7,00}\right) - (27+12\Phi_\kappa)\sum_{k=0}^{2}\left(\frac{7}{2}\beta_{k+1,00} - \frac{7}{22}\beta_{k+4,00} - \frac{175}{286}\beta_{k+7,00}\right)\Bigg] + \frac{21}{8}\left(\frac{3+4\Phi_\mu}{1+\Phi_\mu}\right)\sum_{k=0}^{2} a_k(\beta_{k+1,00} + \beta_{k+4,00} + \beta_{k+7,00})\Bigg\} + a_0\beta_{0,11} + a_1\beta_{1,11} + a_2\beta_{2,11}$$

$$c_0 = \frac{309 + 156\Phi_\kappa + \sqrt{(19\,281 - 5232\Phi_\kappa + 5856\Phi_\kappa^2)}}{20 + 4\Phi_\kappa}$$

$$c_1 = \frac{309 + 156\Phi_\kappa - \sqrt{(49\,281 - 5232\Phi_\kappa + 5856\Phi_\kappa^2)}}{20 + 4\Phi_\kappa}.$$

$$\hat{D}_0(\tau) = \beta_{5,21}\,e^{-a_0\tau} + \beta_{6,21}\,e^{-a_1\tau} + \beta_{7,21}\,e^{-a_2\tau} + \beta_{8,21}\,e^{-d_0\tau} + \beta_{9,21}\,e^{-d_1\tau} \quad \text{(B5)}$$

$$\hat{D}_1(\tau) = \beta_{0,21}\,e^{-a_0\tau}+\beta_{1,21}\,e^{-a_1\tau}+\beta_{2,21}\,e^{-a_2\tau}+\beta_{3,21}\,e^{-d_0\tau}+\beta_{4,21}\,e^{-d_1\tau} \quad \text{(B6)}$$

with the constants $\beta_{k,21}$ defined as

$$\beta_{k,21} = \left\{\frac{1}{\Phi_\alpha}[a_k(135+54\Phi_\kappa)-693(2+3\Phi_\kappa)][143\beta_{k+1,00} - 13\beta_{k+4,00}-25\beta_{k+7,00}] - \frac{1}{\Phi_\alpha}[a_k(13+6\Phi_\kappa) - 63(2+3\Phi_\kappa)]\left[1287\beta_{k+1,00}+99\beta_{k+4,00}-\frac{693}{5}\beta_{k+7,00}\right] + \frac{429a_k^2}{4}[\beta_{k+1,00}+\beta_{k+4,00} + \beta_{k+7,00}]\right\}\left[-\frac{W\Delta_{k,21}}{1+\Phi_\mu}\right], \quad k=0,1,2$$

$$\beta_{3,21} = \frac{d_1\gamma_5+\gamma_6}{d_1-d_0}, \quad \beta_{4,21} = \frac{d_0\gamma_5+\gamma_6}{d_0-d_1}$$

$$\beta_{k+5,21} = \left[\frac{12a_k}{1001}\left(\frac{13+3\Phi_\kappa}{2+3\Phi_\kappa}\right)-\frac{58+33\Phi_\kappa}{7(2+3\Phi_\kappa)}\right]\beta_{k,21} + \frac{W}{126\Phi_\alpha(1+\Phi_\mu)}\left\{\frac{8505}{8}\left(\frac{5+2\Phi_\kappa}{2+3\Phi_\kappa}\right)\left[\frac{8}{315}\beta_{k+1,00} - \frac{8}{3465}\beta_{k+4,00}-\frac{40}{9009}\beta_{k+7,00}\right] - \frac{6237}{8}\left(\frac{13+6\Phi_\kappa}{2+3\Phi_\kappa}\right)\left[\frac{8}{693}\beta_{k+1,00} + \frac{8}{9009}\beta_{k+4,00}-\frac{8}{6435}\beta_{k+7,00}\right]+\frac{3\Phi_\alpha}{4(2+3\Phi_\kappa)}a_k[\beta_{k+1,00} + \beta_{k+4,00}+\beta_{k+7,00}]\right\}, \quad k=0,1,2$$

$$\beta_{8,21} = \left[\frac{12d_0}{1001}\left(\frac{13+3\Phi_\kappa}{2+3\Phi_\kappa}\right)-\frac{58+33\Phi_\kappa}{7(2+3\Phi_\kappa)}\right]\beta_{3,21}$$

$$\beta_{9,21} = \left[\frac{12d_1}{1001}\left(\frac{13+3\Phi_\kappa}{2+3\Phi_\kappa}\right)-\frac{58+33\Phi_\kappa}{7(2+3\Phi_\kappa)}\right]\beta_{4,21}$$

where

$$\Delta_{k,21} = (2808+648\Phi_\kappa)a_k^2-(146\,520+80\,784\Phi_\kappa)a_\kappa + 648\,648(2+3\Phi_\kappa), \quad k=0,1,2$$

$$\gamma_5 = \frac{-W}{1+\Phi_\mu}\left[\frac{429\Phi_\kappa}{64(13+3\Phi_\kappa)(27+2\Phi_\kappa)}\right]-\beta_{0,21}-\beta_{1,21}-\beta_{2,21}$$

$$\gamma_6 = \frac{1573W}{576(1+\Phi_\mu)}\left[\frac{(1665+918\Phi_\kappa)\Phi_\kappa}{(27+2\Phi_\kappa)(13+3\Phi_\kappa)^2}\right] + \frac{143W}{216\Phi_\alpha(1+\Phi_\mu)}\left\{\frac{35}{16}\left(\frac{2430+972\Phi_\kappa}{13+3\Phi_\kappa}\right)\sum_{k=0}^{2}\left[\frac{8}{315}\beta_{k+1,00} - \frac{8}{3465}\beta_{k+4,00}-\frac{40}{9009}\beta_{k+7,00}\right] - \frac{6237}{16}\left(\frac{26+12\Phi_\kappa}{13+3\Phi_\kappa}\right)\sum_{k=0}^{2}\left[\frac{8}{693}\beta_{k+1,00} + \frac{8}{9009}\beta_{k+4,00}-\frac{8}{6435}\beta_{k+7,00}\right] + \frac{3\Phi_\alpha}{4(13+3\Phi_\kappa)}\sum_{k=0}^{2}a_k[\beta_{k+1,00}+\beta_{k+4,00}+\beta_{k+7,00}]\right\} + a_0\beta_{0,21}+a_1\beta_{1,21}+a_2\beta_{2,21}$$

$$d_0 = \frac{11(185+102\Phi_\kappa)+\sqrt{(1\,330\,417-298\,320\Phi_\kappa+285\,912\Phi_\kappa^2)}}{39+9\Phi_\kappa}$$

$$d_1 = \frac{11(185+102\Phi_\kappa)-\sqrt{(1\,330\,417-298\,320\Phi_\kappa+285\,912\Phi_\kappa^2)}}{39+9\Phi_\kappa}.$$

TRANSFERT THERMIQUE CONJUGUE POUR UNE GOUTTE EN TRANSLATION A FAIBLE NOMBRE DE PECLET DANS UN CHAMP ELECTRIQUE

Résumé—On considère l'échange thermique conjugué avec température variable à l'interface entre une goutte liquide et le fluide ambiant dans un champ électrique uniforme. On développe une perturbation singulière pour obtenir la température dans le domaine de phase continue tandis que la perturbation régulière est utilisée pour obtenir la solution dans la goutte avec le secours de la méthode des résidus pondérés. Cette méthode est trouvée puissante pour résoudre les problèmes avec des hétérogénéités dépendant du temps à cause de l'équation et ou des conditions aux limites. La température est calculée au premier ordre du nombre de Peclet; néanmoins un ordre plus élevé est aussi atteint pour la phase ambiante de façon à examiner l'influence d'un champ externe sur les flux totaux transférés. Dans la solution de premier ordre, les effets d'un champ électrique altèrent la température à l'intérieur et à l'extérieur de la gouttelette ainsi que le flux thermique, mais le flux net transféré qui est totalement gouverné par la conduction et la convection demeure inchangé. Au delà de l'approximation du premier ordre, la contribution du transfert net de chaleur à cause du champ électrique devient calculable.

KONJUGIERTER WÄRMEÜBERGANG AN EINEM BEI KLEINER PECLET-ZAHL IN EINEM ELEKTRISCHEN FELD BEWEGTEN TROPFEN

Zusammenfassung—Es wird der konjugierte Wärmeaustausch durch die Grenzfläche zwischen einem bewegten Flüssigkeitstropfen und dem umgebenden Fluid in einem gleichförmigen elektrisichen Feld betrachtet, wobei die Grenzflächentemperatur zeitlich veränderlich ist. Die Temperatur im Gebiet der kontinuierlichen Phase (in der Umgebung des Tropfens) wird mit Hilfe eines singulären Störungsansatzes ermittelt, während ein reguläres Störungsverfahren unter Anwendung der Methode der gewichteten Residuen für die Lösung des Temperaturfeldes im Tropfen benutzt wird. Dieses Verfahren erweist sich als wirksames Instrument bei der Lösung von Problemen mit zeitabhängigen Nicht-Homogenitäten, die sich aus der zu Grunde liegenden Gleichung und/oder den Randbedingungen ergeben. Die Temperatur wird bis einschließlich der Peclet-Zahl erster Ordnung berechnet. Darüberhinaus werden auch höhere Ordnungen für das umgebende Fluid betrachtet, um den Einfluß eines äußeren Feldes auf den Gesamttransport zu untersuchen. Bei der Lösung erster Ordnung wirkt sich ein elektrisches Feld so aus, daß sich die Temperatur innerhalb und außerhalb des Tropfens ändert. Dasselbe gilt für die Wärmestromdichte, wobei jedoch der Nettowärmestrom, der sich aufgrund von Leitung und Konvektion ergibt, unverändert bleibt. Bei Lösungen höherer Ordnung wird der Beitrag zum Nettowärmetransport aufgrund des elektrischen Feldes abschätzbar.

СОПРЯЖЕННЫЙ ТЕПЛОПЕРЕНОС ОТ КАПЛИ, ПЕРЕМЕЩАЮЩЕЙСЯ В ЭЛЕКТРИЧЕСКОМ ПОЛЕ ПРИ НИЗКОМ ЧИСЛЕ ПЕКЛЕ

Аннотация—Исследуется сопряженный теплоперенос при нестационарной температуре на границе раздела, происходящий между падающей каплей и объемом жидкости в условиях действия постоянного электрического поля. Температура сплошной фазы определяется с использованием сингулярного возмущения, в то время как решение для области внутри капли получено методом взвешенных разностей с использованием регулярного возмущения. Этот метод является действенным при решении задач с нестационарными неоднородностями, возникающими в определяющем уравнении и/или в граничных условиях. Температура рассчитывается вплоть до первого порядка числа Пекле; однако с целью установления влияния внешнего поля на скорости суммарного переноса в основной фазе определяются температуры и более высокого порядка. В решении первого порядка эффекты электрического поля вызывали изменение температуры и теплового потока внутри капли и вне ее, а скорость суммарного теплопереноса, определяемая исключительно теплопроводностью и конвекцией, оставалась постоянной. За пределами приближения первого порядка становится возможной оценка вклада электрического поля в суммарный теплоперенос.